RSAC 2026 Conference

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SoK: A Generalized Attack on RSA and Its Variants

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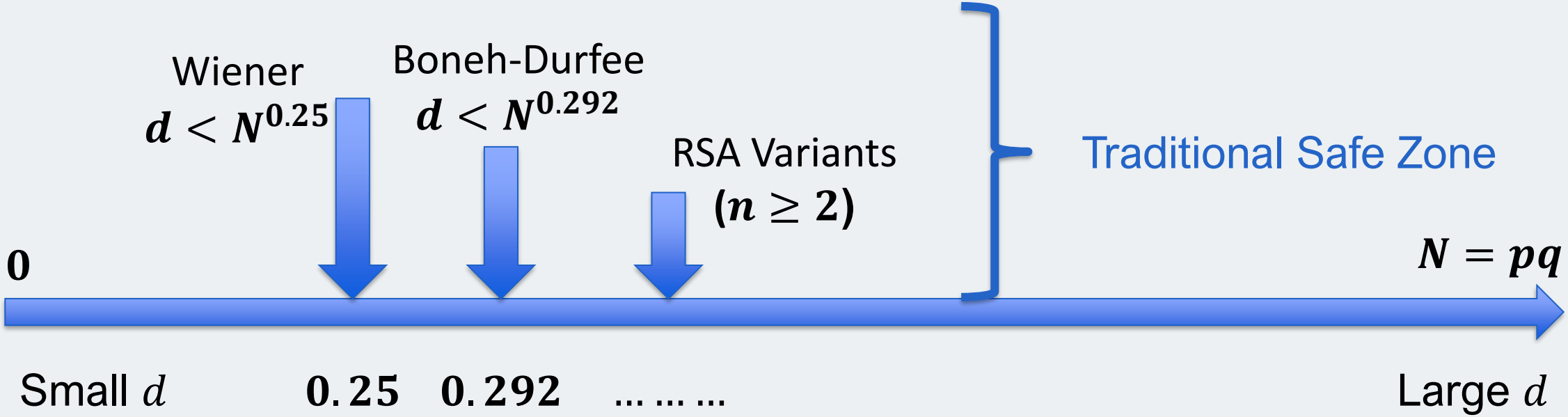
The Status of RSA Security

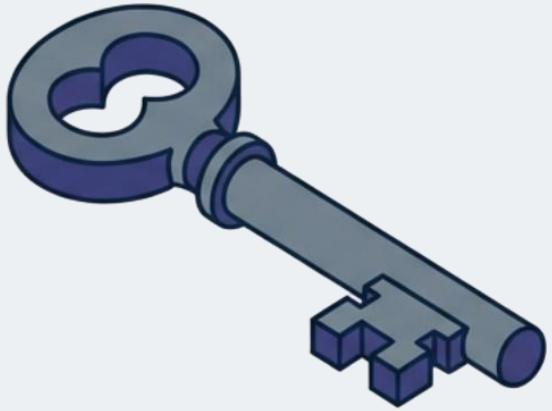


$$ed \equiv 1 \bmod \varphi(N) \Rightarrow ed \equiv 1 \bmod (p-1)(q-1)$$

Current Cryptanalysis: A large private key d ensures security.

Blind Spot: Is Size the Only Vulnerability?

Standard View

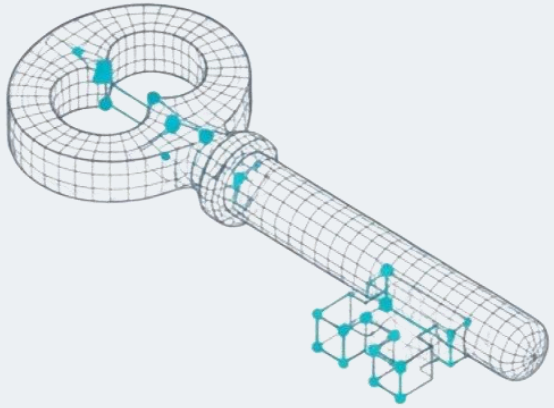


Large d $(d \approx N)$

Could RSA be weak even if d is large and theoretically safe from all known attacks?

Yes!

Generalized View



Hidden Algebraic Perturbations

Redefining RSA's Modular Structure

Standard RSA

$$\varphi(N) = (p-1)(q-1)$$

Known RSA Variants

$$\psi_n(N) = (p^n - 1)(q^n - 1)$$

New Generalized Key Equation

$$\psi_n(N, a, b) = (p^n - a)(q^n - b)$$

Crucial Insight

Traditional functions are merely special instances where $a = b = 1$. By allowing a and b to vary, there exists a massive, previously invisible attack surface.

The Generalized Key Equation

Public Exponent (Known)

Generalized Inverse (Replacing d)

$$eu \equiv 1 \bmod \psi_n(N, a, b)$$

Hidden Structure $(p^n - a)(q^n - b)$

Paradigm Shift: No longer hunting for d, but hunting for u, the inverse with respect to a perturbed structure defined by unknown a and b.

One Framework to Rule Them All

Attack / Research	Fixed Parameters	Insecure Bound (δ)
Boneh-Durfee	$n = 1, a = 1, b = 1$	$\delta < 1 - \sqrt{2}/2 \approx 0.292$
Peng et al. / Zheng et al.	$n = 2, a = 1, b = 1$	$\delta < 2 - \sqrt{\alpha}$
Teleseanu	arbitrary $n, a = 1, b = 1$	$\delta < n - \sqrt{\alpha n/2}$
Our Generalized Attack	arbitrary n, unknown $a, b \geq 1$	**New Attack Surface**

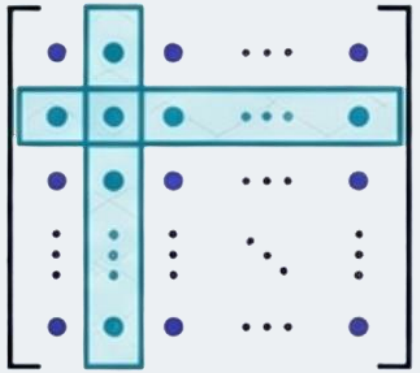
Note: Previous research only explored a single line for $a = b = 1$!

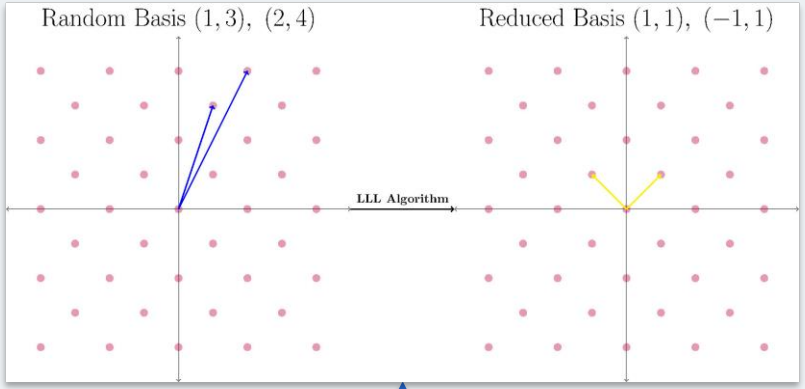
Attack Engine: Coppersmith & Lattices

Target Polynomial → Linearization → Lattice → LLL Reduction

$$eu \equiv 1 \bmod \psi_n(N, a, b)$$
$$\Downarrow$$
$$f(x, y) = x(N^n + y) + 1 \bmod e$$

Herrmann-May technique reduces to the linear form $g(x, z)$ using $z = xy + 1$





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From Approximations to Exact Factors

LLL Output Small Roots

$$y_0 = -(aq^n + bp^n - ab)$$

$$p, q$$

Error term: $|aq^n - bp^n| < 2N^{n/2}$

New Technique: Lemma 1

Step 1: Recover Sum S & Difference D

$$S = aq^n + bp^n \text{ and } D = |aq^n - bp^n|$$

Step 2: Extract Two Primes p & q

$$p, q = \{\gcd(N, S + D), \gcd(N, S - D)\}$$

New Bounds of Vulnerability

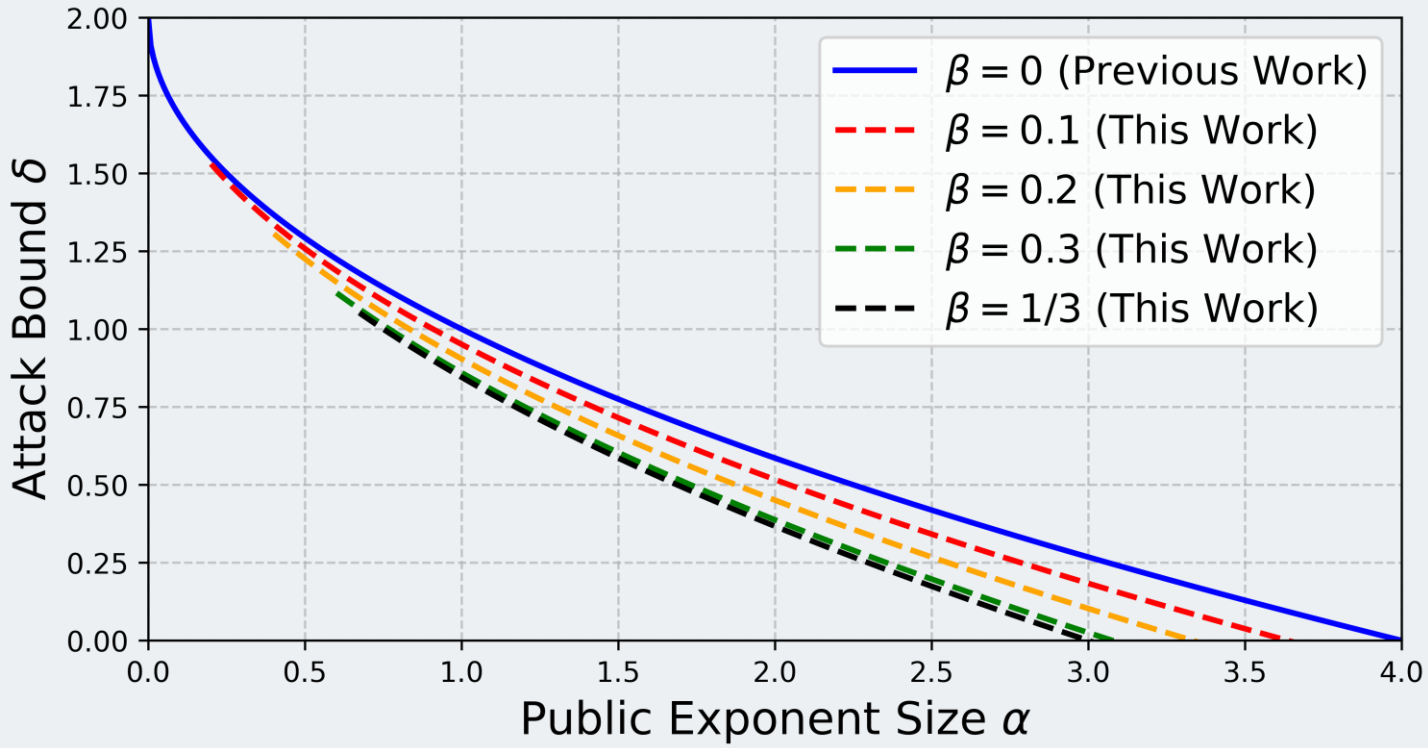
$$\delta < n - \frac{\sqrt{\alpha(2\beta + n)}}{2}$$

- α: Size of Public Key ($e = N^{\alpha}$)
- β: Size of Structural Perturbation ($a, b \leq N^{\beta}$)
- δ: Size of Generalized Inverse ($u < N^{\delta}$)

Constraint: The attack holds if parameters satisfy this bound, provided that

$$\alpha > n/2 + \beta \text{ and } \beta < n/6.$$

Visualizing the Attack Surface (2D)

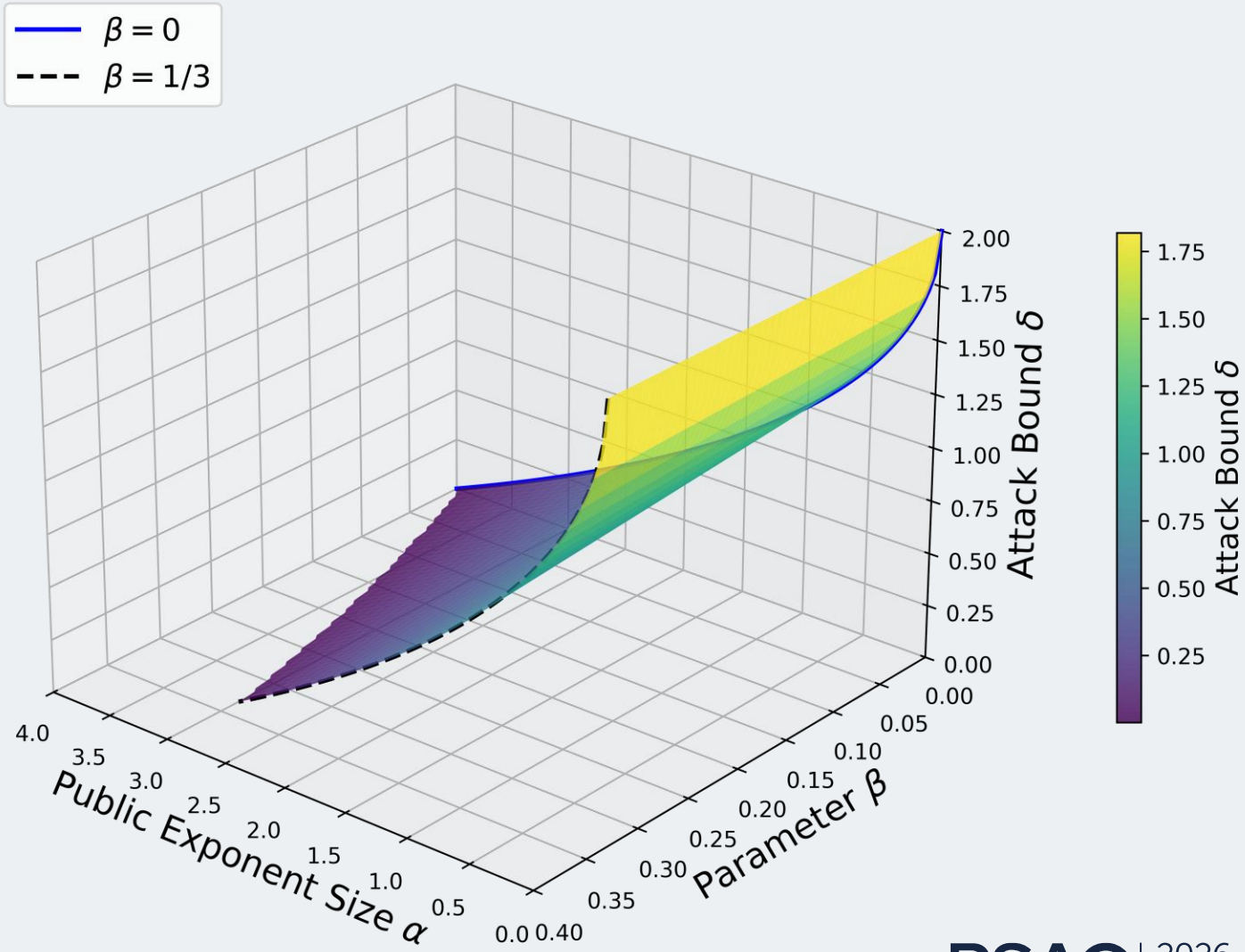


Note: Increasing structural complexity lowers the curve, but reveals a multi-dimensional attack surface previously thought secure.

Visualizing the Attack Surface (3D)

Security Polytope

Security is not a binary switch; it is now a continuous geometric volume.



‘Safe’ Keys That Are Not Safe

The Illusion of Safety

- Modulus: 200-bit N and $n = 2$
- Public Key: 400-bit e ($\alpha \approx 1.988$)
- Target Private Exponent: $d \approx N^{0.988}$

Verdict: Secure against Boneh-Durfee attack and other cryptanalyses.

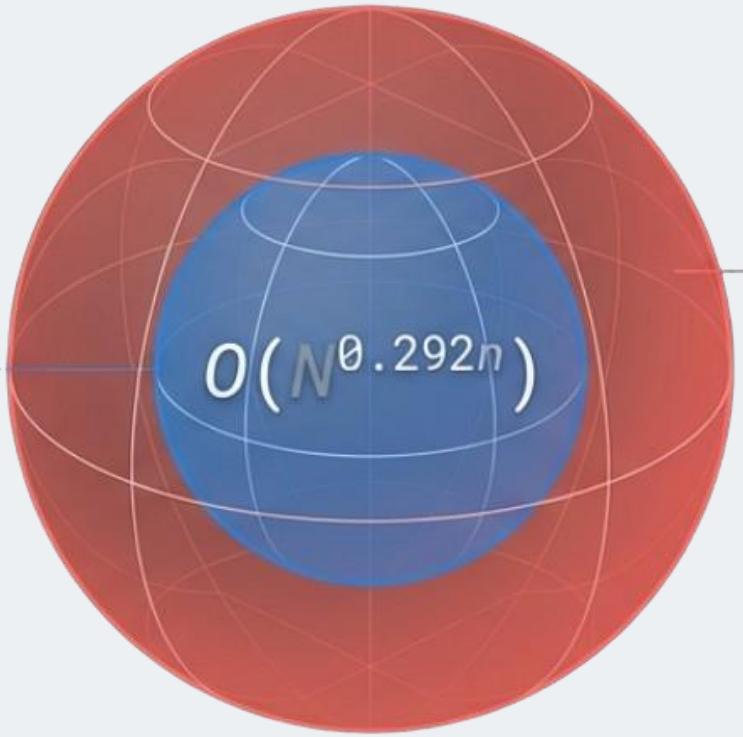
The Reality of Weakness

- Attack Parameters: $\beta = 0.25$ (unknown a, b exist)
- Generalized Inverse: $\delta = 0.35$

Result: Factoring algorithm recovers p and q in seconds.

Expanding the Weak Key Universe

Standard Weak Keys
$\mathcal{O}(N^{0.292n})$



Generalized Weak Keys
$\mathcal{O}(N^{0.350n})$

Note: An RSA instance secure against all previous attacks may still be broken if its public key possesses such hidden algebraic structure.

Apply What Have Learned

- **Analyze** existing public keys. Check for hidden algebraic relations

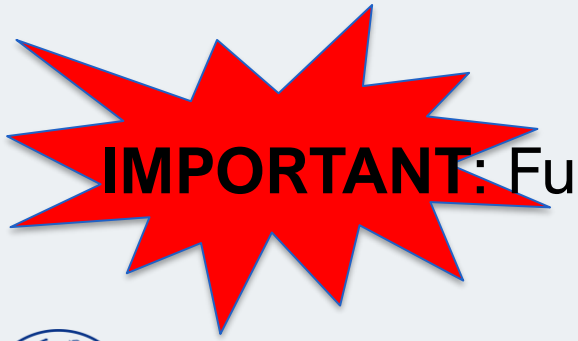
$$eu \equiv 1 \bmod \psi_n(N, a, b).$$

- **Audit** key generation libraries. Ensure entropy sources do not inadvertently create relations with small a and b.
- **Test** some keys using the open-source SageMath implementation provided

https://github.com/MengceZheng/RSA_GAF

Summary of Findings

- **Unified:** Bridged the gap between separate attacks on RSA and its variants into one algebraic framework.
- **Expansive:** Proved that several large private exponents offer no guarantee of safety if hidden structural parameters exist.
- **Constructive:** The proposed attack is practical, using LLL reduction and Lemma 1 to recover prime factors in polynomial time.



Full-size private keys are no longer assumed to ensure security!

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Thank you!

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